15th EAWE PhD Seminar on Wind Energy
29–31 October 2019
Centrale Nantes, France

Set-point optimization in wind farms to mitigate effects of flow blockage induced by gravity waves

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Keywords: gravity waves, flow blockage, set-point optimization, adjoint optimization

1 Introduction

Nowadays, it is well known that turbines strongly interact when clustered together in large arrays, increasing the momentum deficit in the lowest region of the atmospheric boundary-layer. These turbine-turbine interactions, such as reduced wind speed and increased turbulence intensity, occur within the wind-farm area and can lead to detrimental consequences in terms of farm's efficiency. However, it has been recently discovered that also non-local effects such as gravity waves can have strong implications on the wind-farm energy extraction.

Wind-farm induced gravity waves are triggered by the upward displacement of the boundary layer which is caused by the diverging fluid streamlines due to flow deceleration. By using a quasi-analytical model of atmospheric response to wind-farm drag, Smith [1] found out that gravity-wave excitation is strongly dependent upon the height of the boundary layer and the stability of the atmosphere aloft. Later, a fast boundary-layer model was proposed by Allaerts & Meyers [2]. They used it for an annual energy production study of the Belgian-Dutch offshore wind-farm cluster, showing that the annual energy loss due to the effect of self-induced gravity waves may be of the order of 4 to 6 % [3]. The goal of the current study is to asses whether it is possible to mitigate the effects of flow blockage induced by gravity waves by varying the turbine thrust set-point.

2 Methodology

The study is performed using a fast boundary-layer model which divides the vertical structure of the atmosphere in three layers [2]. The wind-farm drag force is applied over the whole wind-farm area and is directly proportional to the thrust set-point distribution $C_T = C_T(x, y, t)$. The goal is to find an optimal control C_T which maximizes the wind farm energy extraction over a period time T. The constraints are the linearized, incompressible, depth-averaged Reynolds-Averaged Navier-Stokes equations with buoyancy term; the box constraint $0 \leq C_T < 1$ is also taken into account.

The input of the optimization model is the initial thrust set-point distribution C_T while the outputs are the optimal turbine thrust coefficient distribution C_T^{opt} and the maximum energy extracted by the wind-farm. The wind-farm layout and the atmospheric conditions affect considerably the gravity waves pattern, hence it is necessary to specify them beforehand.

The governing equations are discretized using a Fourier-Galerkin spectral technique. A pseudo-spectral method is used for computing the first-order term of the wind-farm drag; the aliasing error is removed using the 3/2-rule [4]. The discretized equations form a linear matrix equation which is solved using the LGMRES algorithm. The L-BFGS-B algorithm is used for solving the non-linear PDE-constrained optimization problem; in order to speed up the optimization algorithm, the gradient of the objective function is evaluated by using the continuous adjoint method.

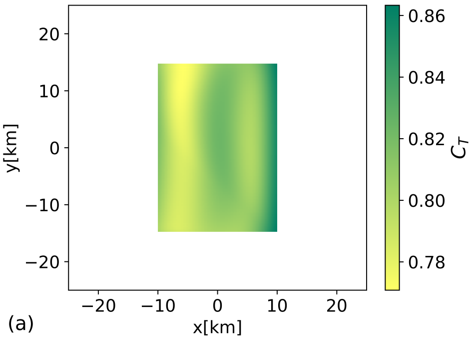

Figure 1: Planform view of (a) optimal turbine thrust coefficient space distribution in sub-critical condition ($F_r = 0.9$) and (b) in super-critical condition ($F_r = 1.1$). The length and width of the wind-farm is 20 and 30 km, respectively.

3 Results and discussion

The model parameters correspond to the ones used by Allaerts & Meyers [2] (cf. their Table 4). The optimal thrust set-point distributions obtained for different Froude numbers are shown in Fig.1. Interestingly, we find that the turbine-level optimal setting is non-uniform in space and assumes different spatial distributions according to the atmospheric conditions. In particular, when the flow is sub-critical the optimal wind turbine thrust set-point assumes a sinusoidal behaviour in the streamwise direction while it becomes a U-shaped curve when the flow is super-critical. In both cases, the thrust set-point distribution is almost invariant along the spanwise direction.
The patterns of these spatial distributions are determined by the vertical displacement of the inversion layer above the wind-farm (not shown) which triggers gravity waves; by reducing the wind turbine thrust set-point in the regions shown in Fig.1, the displacement of the inversion layer is reduced as well, hence the strength of the pressure gradient induced by gravity waves decreases. Consequently, the farm-averaged velocity increases which leads to an increment of 1.75 % and 2.30 % in wind farm energy extraction for the sub-critical and super-critical flow, respectively. The energy extraction gains are referred to a wind-farm which uses a uniform thrust set-point distribution, with $C_T = C_T^{\text{Betz}}$.
Regarding time-dependency of C_T^{opt}, results obtained (not shown) indicate that the optimal thrust set-point distribution is time-independent, meaning that it is not necessary to excite a non-stationary wave behaviour in order to further increase the wind-farm energy extraction.

4 Future work

The optimal thrust set-point distribution and the relative energy extraction gain are strongly dependent upon the atmospheric conditions. It would be useful to conduct a sensitivity study in order to asses the cases in which the energy extraction gain is substantial and the ones in which it is negligible. Moreover, the representation of the wind-farm drag force can be improved by using a Gaussian wake model [2, 5]. Finally, it would be useful to study the gravity-wave feedback with a Large Eddy Simulation (LES) in order to validate the results discussed in the previous section.

References

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