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On the Use of Nonlinear Normal Modes in Model Reduction

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1 Introduction

In the wind energy sector, finite element (FE) modelling plays an essential role in the design process and monitoring of assets. With the increasing technical demands being placed on wind turbines, the use of high fidelity models has become mainstream, especially with regards to composite and flexible components such as blades. Of particular interest for new designs, involving taller and more slender towers, and higher blade spans, are numerical models that can account for nonlinear effects. However, with the increased accuracy of these models comes a corollary increase in the computational effort required for their analysis. This hinders verification and validation (VV) processes, which typically involve model updating, uncertainty quantification and real-time simulation, whereby the compuational cost can be prohibitive. This has motivated the development of model order reduction methods, which seek to approximate the behaviour of a full model to a required degree of precision but with greatly decreased computational cost. Although corresponding established methods for linear systems are already broadly established, when considering nonlinear systems these methods can break down.

Linear model order reduction techniques such as the Craig-Bampton and MacNeal-Rubin methods [1],[2] are well developed and widely used for the reduction of linear FE models. These methods largely make use of linear normal modes as a reduced basis on which the systems' equations of motion are projected. Normal modes form a very efficient reduction basis for linear systems due to their orthogonality and the ability to target certain frequency regions of interest. When considering nonlinear systems however, these linear methods are not appropriate as the concept of linear normal modes no longer holds. The most common method for the reduction of weakly nonlinear FE models adopts Proper Orthogonal Decomposition (POD), wherein a dynamic simulation of the system is carried out with"snapshots" of the displacement field extracted at various time intervals [3]. These snapshots are used to construct an optimal linear reduction basis using singular value decomposition [4]. Related recent work explores the extension of the traditional reduction approach by inclusion of higher order enrichments, such as mode shape derivatives [5].

An alternative to this reduction approach is offered by, a theory which extends the concept of normal mode shapes to nonlinear systems, the so-called nonlinear normal modes (NNMs). There are several different formulations of NNMS with the Shaw-Pierre modes being considered in this work which build on the fundamental work of Rosenberg [6]. An NNM is considered to be "a motion which takes place on a two dimensional manifold in phase-space" [7], meaning that within an NNM the motion of each point in the system can be given as a function of the displacement and velocity of a single point. These NNMs have the potential to provide a very efficient reduction basis for lightly damped nonlinear systems as they maintain the property of modal invariance of linear normal modes, whereby a motion in one mode will not effect motion in any of the other modes [7]. The derivation of these NNMs is usually carried out by assuming a power series solution for the nonlinear manifold functions which can then be solved analytically for limited simple cases or numerically for more complex systems [7]. Recent work however, has shown how machine learning methods, such as locally linear embedding and autoencoder neural networks can be used to extract NNMs from output only data [8]. In this work autoencoder neural networks are used to extract NNMs from nonlinear system data and used to form a reduction basis.

2 Autoencoder Neural Networks

Autoencoders typically five layer neural networks often used for dimensionality reduction or de-noising problems [9]. They are constructed, as illustrated in figure 1, as a deep neural network architecture with a so-called "bottleneck" layer in which the number of nodes is reduced, in the case of this example to two, although the size of this bottleneck layer can be tuned such that a threshold amount of variance is preserved. The key concept of their operation involves setting the cost function of the network such that the output attempts to re-create the inputs as closely as possible. By the inclusion of the bottleneck layer, which forces the data through a lower dimensional feature space, a near optimal reduction of the data onto the feature space dimension can be achieved which preserves as much information pertaining to the response as possible. The reduced dimension feature space in the bottleneck layer is considered to be the NNMs of the system.

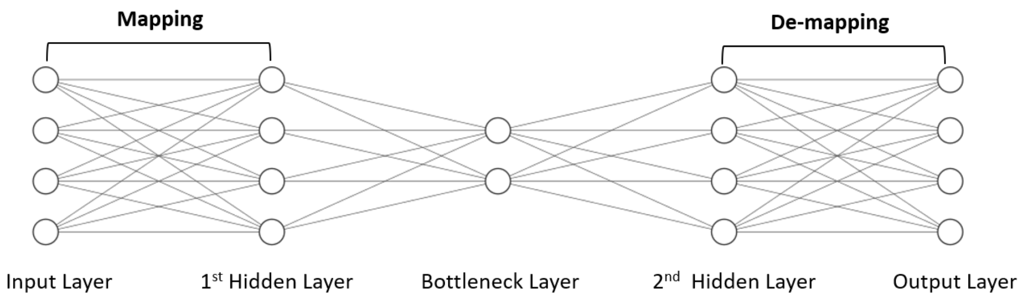
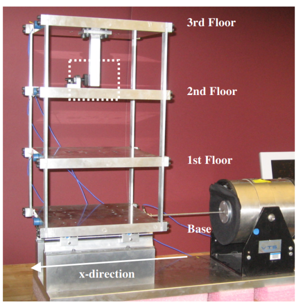


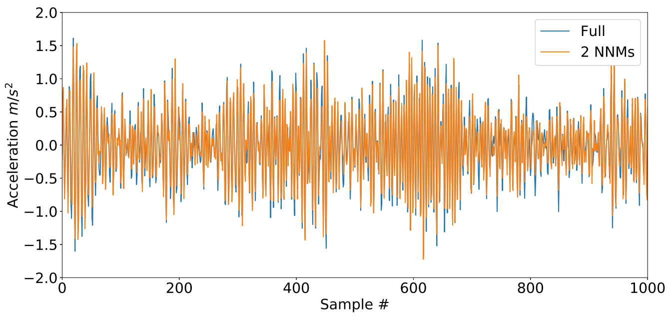
Figure 1: Architecture of an AANN with input/output dimension of 4 and a bottleneck dimension of 2.

3 Example

As a brief demonstration of this method, an example is made in which 2 NNMs are extracted from output only measurements from a 4DOF nonlinear frame structure tested experimentally [10]. The frame structure was excited with white noise and acceleration measurements taken at each of the four floors. Figure 2 demonstrates that the response can be well approximated using only 2 NNMs extracted from the data as opposed to the original 4 dimensions of the data.



(a)



(b)

Figure 2: (a) the experimental rig tested, (b) the response preserved from 2 NNMs

4 Extension To Wind Turbine Blades

Future work will extend the use of this method for extracting NNMs to a high fidelity wind turbine blade model with geometric nonlinearity. These NNMs will be used to form a reduced basis of the model and compared in performance to traditional reduction methods such as POD. The reduced basis formed will be used to create a reduced order model of the system which will be compared to the full solution for speed of computation and accuracy.

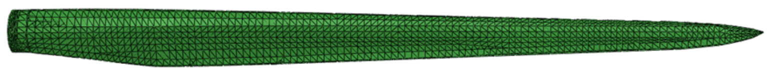


Figure 3: Wind turbine blade FE model

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